Topic: Solving with factoring

Question: Find the limit.

$$\lim_{t \to -1} \frac{t^3 + 1}{t + 1}$$

Answer choices:

A 0

B 3

C -1

D ∞

© Krista King Math

Solution: B

The limit of

$$f(t) = \frac{t^3 + 1}{t + 1}$$

as x approaches -1 is 3. Even though the denominator is zero when we try to evaluate by direct substitution, we have a clue that the zero will divide out of the denominator because we get $0/0$ by direct substitution. So we factor the numerator using long division of polynomials to see that the zero does in fact divide out of the denominator.

$$f(t) = \frac{t^3 + 1}{t + 1}$$

$$f(t) = \frac{(t + 1)(t^2 - t + 1)}{t + 1}$$

$$f(t) = t^2 - t + 1$$

Since

$$\frac{t^3 + 1}{t + 1} = t^2 - t + 1$$

for all values of t except $t = -1$, the two functions have the same limit at $t = -1$, which can be found by direct substitution into the equivalent equation.

$$g(t) = t^2 - t + 1$$

$$g(-1) = (-1)^2 - (-1) + 1$$

© Krista King Math

$$g(-1) = 1 + 1 + 1$$

$$g(-1) = 3$$

© Krista King Math

Topic: Solving with factoring

Question: Evaluate the limit.

$$\lim_{x \to 2} \frac{x^2 - 2x}{x - 2}$$

Answer choices:

A 4

B -4

C 2

D -2

© Krista King Math

Solution: C

To solve a limit by factoring, factor the numerator and denominator of the given rational function. The goal is to find a common factor in both the numerator and the denominator, so that we can cancel it.

We'll start with direct substitution.

$$\lim_{x\to 2} \frac{x^2-2x}{x-2} = \frac{(2)^2-2(2)}{(2)-2}$$

$$\lim_{x\to 2} \frac{x^2-2x}{x-2} = \frac{4-4}{2-2}$$

$$\lim_{x\to 2} \frac{x^2-2x}{x-2} = \frac{0}{0}$$

Because we get an indeterminate form, we'll try factoring instead.

$$\lim_{x\to 2} \frac{x^2-2x}{x-2}$$

$$\lim_{x\to 2} \frac{x(x-2)}{x-2}$$

Cancel common factors.

$$\lim_{x\to 2} x$$

Now we can try substituting again.

$$2$$

This is the value of the limit.

© Krista King Math

Topic: Solving with factoring

Question: Evaluate the limit.

$$\lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 - 9}$$

Answer choices:

A $\frac{1}{3}$

B $-\frac{1}{3}$

C $\frac{1}{6}$

D $-\frac{1}{6}$

© Krista King Math

Solution: D

To solve a limit by factoring, factor the numerator and denominator of the given rational function. The goal is to find a common factor in both the numerator and the denominator, so that we can cancel it.

We'll start with direct substitution.

$$\lim_{x\to 3} \frac{x^2-7x+12}{x^2-9} = \frac{(3)^2-7(3)+12}{(3)^2-9}$$

$$\lim_{x\to 3} \frac{x^2-7x+12}{x^2-9} = \frac{9-21+12}{9-9}$$

$$\lim_{x\to 3} \frac{x^2-7x+12}{x^2-9} = \frac{0}{0}$$

Because we get an indeterminate form, we'll try factoring instead.

$$\lim_{x\to 3} \frac{x^2-7x+12}{x^2-9}$$

$$\lim_{x\to 3} \frac{(x-3)(x-4)}{(x-3)(x+3)}$$

Cancel common factors.

$$\lim_{x\to 3} \frac{x-4}{x+3}$$

Now we can try substituting again.

$$\frac{3-4}{3+3}$$

© Krista King Math

$$-\frac{1}{6}$$

This is the value of the limit.

© Krista King Math